

1.8: Operations on Functions

Basic Operations: Given two functions $f(x)$ and $g(x)$ you can create new functions using the basic operations on real numbers. $(+, -, \cdot, \div)$

Examples: Suppose $f(x) = 5x - 1$, $g(x) = 3x + 2$ and $h(x) = x^2 + 8$. Find the following functions.

- $f + g(x)$
 - $h - f(x)$
 - $f \cdot g(x)$
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Composition of Functions: Given two function $f(x)$ and $g(x)$ you can create a new function by putting the output of one function as the input of the second function. This is the idea behind composition of functions.

Definition: For two function $f(x)$ and $g(x)$, the composite function $f \circ g$, also called the composition of f with g is defined by

$$f \circ g(x) = f(g(x)).$$

Example 1: If $f(t) = t^2$ and $g(t) = t + 2$, find

- (a) $f(t + 1)$
- (b) $f(t + h)$
- (c) $f(g(t))$
- (d) $g(f(t))$

Example 2: If $f(x) = e^x$ and $g(x) = 5x + 1$, find

- (a) $f \circ g(x)$
- (b) $g \circ f(x)$

Example 3: Using the following table, find $f \circ g(0)$, $g \circ f(0)$, $f(g(1))$, $f(f(0))$, and $g(f(1))$.

| | | | | |
|--------|---|---|----|----|
| x | 0 | 1 | 2 | 3 |
| $f(x)$ | 3 | 1 | -1 | -3 |
| $g(x)$ | 0 | 2 | 4 | 6 |

Example 4: Use a new variable u for the inside functions to express each of the following as a composite function:

(a) $y = \ln(3t)$

(b) $w = 5(2x + 3)^2$

(c) $P = e^{-0.03t}$